Nucleon Electric Dipole Moments from QCD Sum Rules

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Abstract

The electric dipole moments of nucleons (NEDM, d_N) are calculated using the method of QCD sum rules. Our calculations are based on the parity (P) and time reversal (T) violating parameter $\bar{\theta}$ in QCD and establish a functional dependence of the NEDM on $\bar{\theta}$, without assuming a perturbative expansion of this symmetry breaking parameter. The results obtained from the QCD sum rules approach are shown to be consistent with the general symmetry constraints on CP violations in QCD, including the necessity of: (1) finite quark masses, (2) spontaneous chiral symmetry breaking, and (3) the $U_A(1)$ anomaly. Given the current experimental upper bound on the neutron electric dipole moment (nEDM), $|d_n| \leq 10^{-25} e \cdot cm$, we find $|\bar{\theta}| \leq 10^{-9}$. This result is compatible with previous calculations of nEDM using different techniques and excludes the possibility of solving the strong CP problem within QCD via a dynamical suppression mechanism.

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I. MOTIVATION

In this paper, we study the electric dipole moments of nucleons (NEDM, denoted as d_N), which serves as an indicator of both parity (P) and time reversal (T) symmetry breaking [1]. The main focus is on the possible violations of P and T (or CP) symmetries in the strong interactions, with Quantum Chromodynamics (QCD) as the underlying theory. In this picture, the nucleons (N) are treated as composite particles consisting of quarks (q) and gluons (G) and the strong P and T violating interaction in the QCD Lagrangian is characterized by a $\bar{\theta}$ parameter [2] (see Sec.II for a definition of this parameter). Such a P and T violating interaction among quarks and gluons generates a coupling of the nucleon spin to the external electric field, and the strength of this coupling is defined as the electric dipole moment of the nucleon.

Our purpose is to establish a functional dependence of the NEDM on the $\bar{\theta}$ parameter, along with other fundamental parameters of the QCD Lagrangian, e.g., the quark masses m_q, and the values of quark condensates R_q. Based on the current experimental upper bound of the neutron electric dipole moment (nEDM, denoted as d_n), which is less than $10^{-25} e \cdot cm$ [3], we can obtain an upper bound on the strong CP violating[1] parameter $\bar{\theta}$. Previous calculations, based on effective models of QCD, require $|\bar{\theta}| \leq 10^{-9}$ [4]. The puzzle of explaining such an unnaturally small number is referred to as the strong CP problem [5].

The problem is difficult and interesting, since an analytical calculation of low energy hadronic observables based on the QCD Lagrangian is not a trivial task. Furthermore, it turns out that an important property of QCD, namely – chiral symmetry, is closely related to the strong CP problem and imposes three stringent constraints on the possible breaking of P and T symmetries in QCD, which include: (1) the necessity of non–zero quark masses [6], (2) spontaneous chiral symmetry breaking [7], and (3) the $U_A(1)$ anomaly [8] [9]. These symmetry constraints not only dictate the functional dependence of all CP violating observables on the QCD parameters, but also provide a dynamical suppression to the CP violating observables [9].

A natural solution of the strong CP problem can be obtained without invoking a tiny $\bar{\theta}$ parameter if the dynamical suppression is sufficient to diminish the CP violating observables below the experimental upper bound. Thus, we need to face the challenge how to realize these constraints explicitly in our calculations without employing a perturbative expansion on the $\bar{\theta}$ parameter. Our calculations on the NEDM problem, which is the first one based on the quark–gluon degrees of freedom and the QCD Lagrangian with a $\bar{\theta}$ parameter, will provide a critical answer to such an interesting scenario for the strong CP problem.

This paper is organized as follows: An introduction to the NEDM problem and the strong CP violation is given in section II, where we also set up the notations used in this work. The hadronic and quark–gluon representations of the nucleon correlation functions (NCF) are discussed in section III and section IV, respectively. The results obtained from both representations of the NCF are used to derive QCD sum rules, which are analyzed in section V and section VI. We conclude with a brief summary in section VII.

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[1]Here we use the CPT theorem to translate time reversal noninvariance as CP violation.

II. INTRODUCTION

Before we discuss the QCD sum rule calculations of the nucleon electric dipole moments [10], it is useful to clarify some issues related to the strong CP violation in QCD.

1. CP Violations in QCD

Up to dimension four, the most general QCD Lagrangian in four dimensional space-time, consistent with Lorentz invariance, hermiticity, gauge invariance, is

$$\mathcal{L}_{QCD} \equiv \bar{\psi}\ i\not{D}\psi + m_q\bar{\psi}\ e^{i\theta_q\gamma_5}\psi + \frac{1}{4}G^2 + \frac{g_s^2\theta_G}{32\pi^2}G\tilde{G} \tag{1}$$

where
$$\not{D} \equiv (\partial_\mu + ig_sB_\mu^a\frac{\lambda^a}{2})\cdot\gamma^\mu \tag{2}$$

and
$$\tilde{G}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}, \qquad \epsilon_{0123} = 1 \tag{3}$$

The meanings of various symbols are:

ψ : quark field

$\bar{\psi}$: Dirac adjoint of the quark field, $\bar{\psi} \equiv \psi^\dagger\gamma_0$

B_μ^a : gluon field, $a = 1, .., 8$

$\dfrac{\lambda^a}{2}$: generators of the color $SU(3)$ gauge group, $a = 1, .., 8$

$G_{\mu\nu}$: gluonic tensor field, $G_{\mu\nu} \equiv [\ \partial_\mu + ig_sB_\mu,\ \partial_\nu + ig_sB_\nu\ ],\ G^2 \equiv G_{\mu\nu}G^{\mu\nu}$

g_s : strong coupling constant in QCD

θ_q : quark chiral phase

θ_G : gluon chiral phase

Here we have two P and CP violating (but C even) terms in the QCD Lagrangian,

$$im_q\sin\theta_q\ \bar{\psi}\gamma_5\psi, \qquad \frac{g_s^2\theta_G}{32\pi^2}G\tilde{G}$$

the former is referred to as a quark pseudo–mass term, the second is referred to as a gluon anomaly term. Our notation is chosen such that the corresponding CP violating parameters θ_q and θ_G are angular variables[2] and the QCD generating functional is periodic with respect to these CP violating parameters. The general QCD Lagrangians with two chiral phases θ_q, θ_G are not all physically independent. Through the $U_A(1)$ anomaly [11] [12], we can shift some part of θ_q to θ_G, and vice versa, by performing an $U_A(1)$ rotation on the quark field

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[2]The gluon anomaly term, when evaluated with instanton configurations, gives integer values. Thus, the QCD generating functional is a periodic function of θ_G.

$$\psi \rightarrow \psi' \equiv e^{i\theta\gamma_5}\psi, \qquad\qquad \psi'^{\dagger} = \psi^{\dagger}e^{i\theta\gamma_5}, \qquad\qquad (4)$$

$$\theta_q \rightarrow \theta_q - 2\theta, \qquad\qquad\qquad \theta_G \rightarrow \theta_G - 2\theta. \qquad\qquad (5)$$

Therefore, only the difference between these two phases,

$$\bar{\theta} \equiv \theta_G - \theta_q \qquad\qquad (6)$$

which is invariant under an $U_A(1)$ rotation, is a physical parameter and can be used to label the equivalent classes of CP violating QCD Lagrangians [9]. Furthermore, since physical observables should be independent of the reparameterization of the Lagrangian, we conclude that a CP violating observable should only be proportional to $\bar{\theta}$, instead of being an arbitrary function of θ_q and/or θ_G.

In addition to the explicit symmetry-breaking parameter $\bar{\theta}$, there are three important symmetry constraints which could suppress the magnitude of a CP violating observable. These constraints include: (1) explicit chiral symmetry breaking due to finite current quark masses, (2) spontaneous chiral symmetry breaking with a nonzero quark condensate, and (3) the $U_A(1)$ anomaly [9]. Suppression of the CP violating observables through the symmetry constraints is possible because in any of these particular limits, the effects of strong CP violation vanish, even with a nonzero $\bar{\theta}$ parameter. Therefore, it is desirable to have a calculation of a CP violating observable, e.g., d_N, without assuming a perturbative expansion in the $\bar{\theta}$ parameter. Such a calculation could provide an answer to the important question: Why is strong CP violation small?

2. **Nucleon Electric Dipole Moment as an EM form factor**

In a nondegenerate system like the neutron, the existence of an EDM implies the violation of both parity (P) and time reversal (T, or CP) symmetries. To establish a connection between the CP violating parameter $\bar{\theta}$ in QCD and the NEDM, it is useful to study the nucleon EM matrix element:

$$\begin{aligned} V_{\mu}^{N}(q; p_1, p_2) &\equiv \int d^{2\omega}x \; e^{iqx} \; \langle N(p_2)|J_{\mu}(x)|N(p_1)\rangle \\ &= (2\pi)^{2\omega}\delta^{2\omega}(q - p) \; \langle N(p_2)|J_{\mu}(0)|N(p_1)\rangle \\ &\equiv (2\pi)^{2\omega}\delta^{2\omega}(q - p) \; V_{\mu}^{N}(p_2, p_1) \qquad (7) \end{aligned}$$

$$\text{with} \qquad p \equiv p_2 - p_1 = q \qquad\qquad (8)$$

and 2ω is the space–time dimension. The nucleon EM vertex is extracted from the nucleon EM matrix element by factoring out the (on–shell) Dirac spinor for the nucleon state:

$$V_{\mu}^{N}(p_2, p_1) = \bar{u}(p_2) \; \Gamma_{\mu}^{N}(p_2, p_1) \; u(p_1) \qquad\qquad (9)$$

Using (1) current conservation $\partial_{\mu}J^{\mu} = 0$ and (2) hermiticity $V_{\mu}^{N}(p_2, p_1) = [\, V_{\mu}^{N}(p_2, p_1) \,]^{\dagger}$, we can write down a general form for the EM vertex $\Gamma_{\mu}^{N}(p_2, p_1)$ of spin $1/2$ on-shell nucleon state:

$$\Gamma_\mu^N(p_2, p_1) = F_1^N(q^2)\gamma_\mu + iF_2^N(q^2)\frac{q^\nu\sigma_{\mu\nu}}{2M_N} - F_3^N(q^2)\frac{q^\nu\sigma_{\mu\nu}\gamma_5}{2M_N} + F_4^N(q^2)(q^2\gamma_\mu - q_\mu\hat{q})\gamma_5$$

$$(10)$$

where we have four form factors F_1^N, F_2^N, F_3^N, and F_4^N characterizing the EM properties of the nucleons. At $q^2 = 0$, they are the various EM moments[3] of the nucleon state:

$$e\, F_1^N(q^2 = 0) = Q_N \text{ (charge)} \tag{11}$$

$$\frac{e}{2M_N}\,[F_1^N(q^2 = 0) + F_2^N(q^2 = 0)] = \mu_N \text{ (magnetic moment)} \tag{12}$$

$$\frac{e}{2M_N}\,[F_3^N(q^2 = 0)] = d_N \text{ (electric dipole moment)} \tag{13}$$

$$\frac{1}{2M_N^2}\,[F_4^N(q^2 = 0)] = a_N \text{ (anapole moment)} \tag{14}$$

It is useful to notice that the tensor structure associated with the anomalous magnetic moment F_2^N ($\sigma_{\mu\nu}$) and that associated with the electric dipole moment F_3^N only differs by a factor $i\gamma_5$. In view of this, we can rewrite these EM form factors in a polar form,

$$F_2^N + iF_3^N\gamma_5 \equiv F_N e^{i\alpha_N\gamma_5} \tag{15}$$

$$(F_N)^2 \equiv (F_2^N)^2 + (F_3^N)^2 \tag{16}$$

$$\tan\alpha_N \equiv \frac{F_3^N}{F_2^N} \tag{17}$$

3. Calculation of Hadronic Matrix Elements from QCD

At first sight, to calculate nucleon EM moments from QCD requires a knowledge of the nucleon wave function in terms of the quark gluon basis and a technique of solving the nontrivial strong coupling dynamics in the low energy region. This is certainly beyond our current ability (except for numerical lattice calculations) and we have to rely on other approaches to avoid these complications. For this purpose, we choose to calculate a nucleon correlation function (NCF) in the presence of external EM fields.

$$\Pi_N(p) \equiv \int d^{2\omega}x\ e^{ipx}\langle\, T\ \eta_N(x)\ \bar{\eta}_N(0)\,\rangle_{\theta_q,\theta_G,F_{\mu\nu}} \tag{18}$$

where the nucleon interpolating field η_N is a composite quark operator carrying the same quantum number as a nucleon. For our calculation, we choose

$$\eta_n \equiv (d^t C\gamma_\mu d)\gamma_5\gamma^\mu u \tag{19}$$

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[3]Due to the use of a constant EM background, the contribution of the anapole moment F_4^N vanishes in our calculations.

as a neutron interpolating field [13]. A similar expression for the proton can be obtained by exchanging u and d quarks.

In this approach, the EM matrix element of nucleons, which describes the response of the nucleon states to the weak external perturbation, can be imbedded in the first order expansion of the NCF with respect to the external field. Since we shall focus on the EM form factors at zero momentum transfer, the EM fields can be taken as constants [14].

To the first order of the electric charge, the NCF $\Pi_N(p)$ can be expanded as

$$\Pi_N(p) \;\equiv\; \Pi_N^{(0)}(p) \;+\; e\Pi_N^{\mu\nu}(p)F_{\mu\nu} \;+\; O(e^2) \tag{20}$$

We shall call $\Pi_N^{(0)}(p)$ a nucleon propagator and $\Pi_N^{\mu\nu}(p)$ a (nucleon) polarization tensor. The former describes the propagation of hadronic states carrying the nucleon quantum numbers, the latter gives the EM vertices of hadronic states with the external fields.

4. Tensor Structures of the Nucleon Correlation Function

It is important to know how to write down a complete set of covariant tensors (these will be referred to as basis tensors, composed of one Lorentz vector p_μ and 16 Dirac matrices) and decompose the NCF in terms of these basis tensors. Such structures come out naturally from both hadronic representations (see the discussions in Sec. III) and QCD calculations (see the discussions in Sec. IV) for the NCF. Furthermore, the QCD sum rules are extracted from the coefficient functions associated with these basis tensors. A correct decomposition will help assure that there are no omissions and redundancies in our calculations.

We find that it is convenient to use a commutation-anticommutation relation analysis [15] to generate all possible independent invariant tensors for the NCF. For the nucleon propagator $\Pi_N^{(0)}(p)$, there are 4 independent tensors:

$$I, \quad \gamma_5, \quad \hat{p} \equiv p_\mu \cdot \gamma^\mu, \quad \hat{p}\gamma_5. \tag{21}$$

For the polarization tensor $\Pi_N^{\mu\nu}(p)$, the tensor basis consists of 8 independent second rank tensor matrices:

$$\sigma^{\mu\nu}, \qquad \sigma^{\mu\nu} \cdot \gamma_5 \tag{22}$$
$$p^\mu\gamma^\nu - p^\nu\gamma^\mu, \qquad (p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5 \tag{23}$$
$$\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta, \qquad \epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta \cdot \gamma_5 \tag{24}$$
$$\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu), \qquad \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5 \tag{25}$$

Given these classifications of the basis tensors for the NCFs, we can define various coefficient functions associated with them.

$$\Pi_N(p) \;\equiv\; \Pi_N^{(0)}(p) \;+\; e\Pi_N^{\mu\nu}(p)F_{\mu\nu} \;+\; O(e^2) \tag{26}$$

$$\Pi_N^{(0)}(p) \;\equiv\; f_1^N(p^2) \cdot \hat{p} \;+\; \tilde{f}_2^N(p^2) \cdot I \;+\; i\tilde{f}_3^N(p^2) \cdot \gamma_5 \tag{27}$$
$$\equiv\; f_1^N(p^2) \cdot \hat{p} \;+\; f_2^N(p^2) \cdot e^{i\phi_N(p^2)\gamma_5} \tag{28}$$

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$$\text{with} \qquad [f_2^N(p^2)]^2 \equiv [\tilde{f}_2^N(p^2)]^2 + [\tilde{f}_3^N(p^2)]^2 \qquad (29)$$

$$\tan\phi_N(p^2) \equiv \frac{\tilde{f}_3^N(p^2)}{\tilde{f}_2^N(p^2)} \qquad (30)$$

$$
\begin{aligned}
\Pi_N^{\mu\nu}(p) \equiv\ & \tilde{g}_1^N(p^2)\ \sigma^{\mu\nu} && +\ \tilde{g}_2^N(p^2)\ i\sigma^{\mu\nu}\gamma_5 \\
+\ & g_3^N(p^2)\ i\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\gamma_5 && +\ g_4^N(p^2)\ i(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5 \\
+\ & g_5^N(p^2)\ \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu) && +\ g_6^N(p^2)\ i\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5
\end{aligned}
$$

$$
\begin{aligned}
\equiv\ & g_1^N(p^2)\ \sigma^{\mu\nu}e^{i\varphi_1^N(p^2)\gamma_5} \\
+\ & g_3^N(p^2)\ i\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\gamma_5 && +\ g_4^N(p^2)\ i(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\gamma_5 \\
+\ & g_2^N(p^2)\ \hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)e^{i\varphi_2^N(p^2)\gamma_5}
\end{aligned}
$$

$$(31)$$

$$\text{with} \qquad [g_1^N(p^2)]^2 \equiv [\tilde{g}_1^N(p^2)]^2 + [\tilde{g}_2^N(p^2)]^2 \qquad (32)$$

$$\tan\varphi_1^N(p^2) \equiv \frac{\tilde{g}_2^N(p^2)}{\tilde{g}_1^N(p^2)} \qquad (33)$$

$$[g_2^N(p^2)]^2 \equiv [g_5^N(p^2)]^2 + [g_6^N(p^2)]^2 \qquad (34)$$

$$\tan\varphi_2^N(p^2) \equiv \frac{g_6^N(p^2)}{g_5^N(p^2)} \qquad (35)$$

Notice that because of charge conjugation symmetry, the coefficient function associated with the $\hat{p}\gamma_5$ tensor in the nucleon propagator and those associated with the $p^\mu\gamma^\nu - p^\nu\gamma^\mu$ and $\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta$ tensors are identically zero.

The discussion in the next two sections will be devoted to the constructions of a hadronic and a quark-gluon parameterizations for all these invariant coefficients.

5. **CP Violating Vacuum Condensates and the Quark-Gluon Chiral Phases**

In the QCD sum rule [16] calculations of the hadron observables, we use an operator product expansion (OPE) to expand a hadronic correlation function in the order of operator dimensions. The nonperturbative (to be precise, nonanalytic in the strong coupling constant) structure of the QCD dynamics is parameterized in terms of various vacuum condensates and the perturbative contributions (Wilson coefficients) can be calculated using Feynman rules. By truncating the OPE series at a certain dimension, we obtain an approximate representation of the hadronic correlation function in terms of QCD parameters. On the other hand, the hadronic observables can be built in the hadronic correlation function by inserting a complete hadronic states and expanding the hadronic correlation function according to the hadron invariant masses. Through a matching between quark-gluon and hadron representations, the values of ground state observables can be extracted. It is crucial that we obtain the values of various vacuum condensates from other sources rather than calculating them within the sum rule method. This becomes a problem if we need to include higher dimensional condensates, which are poorly known. Also, in the external field method, due to the polarization of the QCD vacuum, there appear so-called induced condensates we need to take into

account [10]. If CP is not a good symmetry of QCD, there could be in principle more unknown condensates associated with the CP violating operators, e.g., $i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$ and $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$. However, it is possible to relate these parity doublet condensates ($\langle\bar{q}q\rangle_{\theta_q,\theta_G}$ and $i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$) to the chiral phases we have introduced. Indeed, there is a simple theorem, which can be used to relate quark condensates in the parity doublets:

Theorem 1

$$R_q^2 \equiv \left[\langle\bar{q}q\rangle_{\theta_q,\theta_G}\right]^2 + \left[i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\right]^2 \quad \text{is invariant under } U_A(1) \text{ chiral rotations.}$$

The above theorem implies that the two real number $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$ and $i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$ can be thought of as the coordinates of a two dimensional plane, with R_q defining the radius of a chiral circle generated by the $U_A(1)$ chiral rotations.

$$\langle\bar{q}q\rangle_{\theta_q,\theta_G} \equiv -R_q\cos\theta_G \tag{36}$$

$$i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \equiv -R_q\sin\theta_G \tag{37}$$

For this reason, we shall refer R_q as a chiral radius[4][5]. Further use of this theorem will be discussed in latter section, see Sec.IV.

6. $U_A(1)$ **Chiral Rotations and the Use of the Polar Form in the Sum Rule Calculation**

We have seen in the previous discussions that we can rewrite many variables and/or parameters in the polar form. These include:

(a) the quark (θ_q) and gluon (θ_G) chiral phases in the general CP violating QCD Lagrangian (Eq.(1)),

(b) the anomalous magnetic moment (F_2^N) and the electric dipole moment (F_3^N) of the nucleon (Eq.(15)),

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[4]The value of a chiral radius depends on the quark mass. For light flavor $m_q \leq \Lambda_{QCD}$, spontaneous chiral symmetry breaking implies R_q is a finite positive number in the massless limit.

[5]For more than one light quark flavor, the values of the chiral phase

$$\theta_{Gq} \equiv \arctan\frac{i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}}{\langle\bar{q}q\rangle_{\theta_q,\theta_G}}$$

can be determined from Crewther's condition [17]

$$\sum_q \theta_{Gq} = \theta_G \tag{38}$$

$$m_{q_i} R_{q_i} \sin(\theta_{q_i} - \theta_{Gq_i}) = m_{q_j} R_{q_j} \sin(\theta_{q_j} - \theta_{Gq_j}) \tag{39}$$

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(c) the invariant coefficient functions associated with the tensor basis (Eqs.(27),(28),(31)),

(d) the quark condensates in the parity doublet (Eqs.(36),(37)).

We find it is quite convenient to adopt this convention for the following reasons:

(a) The reparameterization invariance of physical observables under the $U_A(1)$ chiral transformations of the QCD Lagrangian can be maintained throughout our calculation. Thus, there is no need to stick to a particular representation of the QCD Lagrangian.

(b) The polar form allows natural identifications among chirally invariant and/or chirally covariant variables. Those chirally invariant observables depend only on the chirally invariant parameters, e.g., m_q, R_q, and $\bar{\theta}$; the chirally covariant variables change by a constant phase under $U_A(1)$ chiral rotations.

(c) Since the $\bar{\theta}$ parameter appears as an angular variable, one can solve the $\bar{\theta}$ dependence of the CP violating observables without using a perturbative expansion. Furthermore, the periodic structure of the $\bar{\theta}$ dependence comes out automatically due to the polar form.

(d) The symmetry constraints on the strong CP violations in QCD can be made transparent in the sum rule relations if we organize both the OPE series and the hadronic representations in the polar forms. Without an explicit solution to the sum rule relations, one can show that CP violating observables vanish if chiral symmetry is exact in QCD.

III. STUDY OF THE NUCLEON CORRELATION FUNCTION (NCF) FROM THE HADRON DEGREES OF FREEDOM

1. **Basics**

One advantage of the QCD sum rule calculation is to extract ground state observables from a correlation function without knowing the exact wave function of the nucleon state. The price for this convenience is that the interpolating field we choose couples to all possible hadronic excited states with the same quantum number as the nucleon. In addition, the ground state matrix element we are interested in is often accompanied with other excited state contributions to the correlation function. Consequently, the extraction of ground state observables from a NCF is possible only if we can identify and isolate various contributions to the NCF from hadronic states. To achieve this purpose, we insert a complete set of hadronic states in the NCF $\Pi_N(p)$ of the interpolating field η_N. In doing so, we can factorize the correlation function into nucleon spinors and hadronic matrix elements. While the ground state observables, e.g., mass M_N and EM moments F_i^N of a nucleon N, can be specified explicitly, the fine details of the excited state spectrum are smeared out by employing suitable parameterizations [14]. The ground state nucleon observables, together with the excited state parameters

are basic ingredients of a hadronic representation of the NCF. We shall discuss the constructions of hadronic representations of the nucleon propagator and the polarization tensor in the following two subsections.

Since we wish to maintain an $U_A(1)$ reparametrization covariance in our calculations, it is important to keep all chiral phases explicit. In particular, we fix the chiral phases of the physical hadron states (and the QCD vacuum, denoted as Ω) to be zero and allow quark fields to be in any chiral basis. With this point in mind, the definition of the nucleon spinor is given by

$$\langle\, \Omega \mid \eta_N \mid N(\vec{p}, s_N)\, \rangle_{\theta_q, \theta_G} \equiv \lambda_N\, e^{i\frac{\theta_N}{2}\gamma_5}\, u(\vec{p}, s_N) \tag{40}$$

Here the nucleon residue λ_N gives the overlap between the nucleon state N and the interpolating field η_N, the nucleon chiral phase θ_N specifies the quark basis of the QCD Lagrangian. It is convenient to choose an interpolating field which transforms like a quark field under an $U_A(1)$ rotation. For instance, for our chosen neutron interpolating field η_n (Eq.(19))

$$u \to u' = e^{i\frac{\theta_u}{2}\gamma_5}u, \qquad\qquad d \to d' = e^{i\frac{\theta_d}{2}\gamma_5}d \tag{41}$$

$$\eta_n \to \eta_n' = e^{i\frac{\theta_u}{2}\gamma_5}\eta_n, \qquad\qquad \theta_n \to \theta_n' = \theta_n + \theta_u \tag{42}$$

In this case, the nucleon chiral phase θ_N transforms covariantly (changing by a constant phase) and the nucleon residue λ_N stays invariant under an $U_A(1)$ rotation.

2. **Hadronic Representation of the Nucleon Propagator**

For the nucleon propagator $\Pi_N^{(0)}(p)$, we can insert a complete set of hadronic states $\sum_N |N\rangle\langle N| = 1$ between the time-ordered product of the interpolating fields $\eta_N, \bar{\eta}_N$.

$$\Pi_N^{(0)}(p) \equiv \int d^{2\omega}x\ e^{ipx}\ \langle\Omega|T(\eta_N(x), \bar{\eta}_N(0))|\Omega\rangle_{\theta_q, \theta_G} \tag{43}$$

$$\equiv \sum_N \int d^{2\omega}x\ e^{ipx}\ \theta(x_0)\langle\Omega|\eta_N(x)|N\rangle_{\theta_q, \theta_G}\langle N|\bar{\eta}_N(0)|\Omega\rangle_{\theta_q, \theta_G}$$

$$- \theta(-x_0)\langle\Omega|\bar{\eta}_N(0)|\bar{N}\rangle_{\theta_q, \theta_G}\langle\bar{N}|\eta_N(x)|\Omega\rangle_{\theta_q, \theta_G} \tag{44}$$

With the definition of the nucleon spinor in a general quark chiral basis (Eq.(40)) and the standard procedure to simplify the algebra, we obtain a propagator of the nucleon state, with an overall chiral conjugation[6]:

$$e^{i\frac{\theta_N}{2}\gamma_5}\ \frac{\lambda_N^2}{\hat{p} - M_N}\ e^{i\frac{\theta_N}{2}\gamma_5}\ =\ \lambda_N^2\ \left(\frac{\hat{p} + M_N \cdot e^{i\theta_N \gamma_5}}{p^2 - M_N^2} \right) \tag{45}$$

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[6]A chiral conjugation of a two–point Green's function is defined as $G(p) \to U(\theta) \cdot G(p) \cdot U(\theta)$, with $U(\theta) = e^{i\frac{\theta}{2}\gamma_5}$.

Similar expressions can be written for excited states N^* with different residues λ_{N^*} and total masses M_{N^*}.

The contributions of the excited states to the NCF lead to many unknowns in our calculations[7]. Consequently, we shall take a simple parameterization to replace the contributions of all excited states without involving the complete hadronic spectrum. Based on a duality argument, we can identify the total contributions of the excited states to the nucleon propagator as the the leading terms from the quark-gluon calculations, starting from a continuum threshold s_0^N:

$$ Re\ f_i^N(p^2)\ (\text{ continuum }) = \frac{1}{\pi}\ Pr. \int_{s_0^N}^{\infty} \frac{Im\ f_i^N(s)\ (\text{ quark-gluon })}{p^2 - s}\ ds \qquad (46) $$

Here we use a dispersion relation for the invariant coefficient functions f_i^N (See Eqs.(27),(28),(31)) to relate the contributions from two representations and $Pr.$ means principal value of the complex integral. The calculations of the quark-gluon representation of the NCF will be given in the next section.

The hadronic representation of the nucleon propagator $\Pi_N^{(0)}(p)$ is given by the sum of Eq.(45) and Eq.(46), where we have three nucleon variables, λ_N, θ_N, and M_N to be determined from the QCD sum rules.

3. **Hadronic Representation of the Polarization Tensor**

Since the polarization tensor $\Pi_N^{\mu\nu}(p)$ comes from a time-ordered product of the electromagnetic current J_μ and the interpolating fields $\eta_N, \bar{\eta}_N$, we insert two complete sets of hadron states, $\sum_N |N\rangle\langle N| = \sum_{N'} |N'\rangle\langle N'| = 1$.

$$ e\Pi_N^{\mu\nu}(p)F_{\mu\nu} \equiv \int d^{2\omega}x\ e^{ipx} \int d^{2\omega}z\ \langle\ T\mathcal{L}_{int}(z)\eta_N(x)\bar{\eta}_N(0)\ \rangle_{\theta_q,\theta_G} \qquad (47) $$

$$ = \sum_N \sum_{N'} \int d^{2\omega}x\ e^{ipx} \int d^{2\omega}z\ \theta(x-z)\ \theta(z-0)\ \times $$

$$ \times\ \langle\Omega|\eta_N(x)|N\rangle_{\theta_q,\theta_G}\langle N|J_\mu(z)|N'\rangle_{\theta_q,\theta_G}\langle N'|\bar{\eta}_N(0)|\Omega\rangle_{\theta_q,\theta_G}A^\mu(z) + $$

$$ + \text{ time ordering} \qquad (48) $$

where the vector potential A^μ of a constant EM field $F_{\mu\nu}$ is given by

$$ A^\alpha \equiv \frac{1}{2}x_\beta F^{\beta\alpha} \qquad (49) $$

The two insertions are independent. Hence, the polarization tensor $\Pi_N^{\mu\nu}(p)$ contains, in addition to the (1) nucleon ground state EM form factors, (2) ground state to excited

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[7]This problem can be traced back to the choice of an interpolating field for the nucleon state. A perfect choice of an interpolating field is equivalent to the solution of an exact wave function of the nucleon state, which couples only to the nucleon state and has zero overlap with any excited state.

states transitions, and (3) transitions among excited states[8]. We can think of the polarization tensor, in its hadronic representation, as a huge matrix in the Fock space and the three contributions are given in a tabular form (see Table 1):

We shall discuss these contributions to various invariant coefficient functions of the polarization tensor, with the superscript labels $g_{Ni}^{(1)}, g_{Ni}^{(2)}, g_{Ni}^{(3)}$ (i is the index for the independent tensor basis) corresponding to three regions listed above. Thus,

$$g_i^N = g_{Ni}^{(1)} + g_{Ni}^{(2)} + g_{Ni}^{(3)}, \qquad i = 1, 2, ..., 6 \qquad (50)$$

(a) the nucleon ground state EM form factors

If we only take the ground state nucleon from the double sum over hadronic complete sets, the polarization tensor reduces to a product of two (CP conserving) nucleon propagators and the nucleon EM vertex, with an overall $U_A(1)$ chiral conjugation.

$$e\,\Pi_N^{\mu\nu}(p)(\text{ nucleon state }) F_{\mu\nu}$$
$$= \int d^{2\omega}z\; e^{i\frac{\theta_N}{2}\gamma_5}\left[\frac{\lambda_N}{\hat{p}-M_N}\; \langle N|J_\mu(z)|N\rangle_{\theta_q,\theta_G}\; A^\mu(z)\; \frac{\lambda_N^*}{\hat{p}-M_N} \right] e^{i\frac{\theta_N}{2}\gamma_5}$$
$$+ \text{ time ordering} \qquad (51)$$

Substituting all ingredients (Eqs.(10), (40), (49)) into the equation Eq.(51), we obtain the contribution from the ground state nucleon to the polarization tensor (We have factored out an overall constant $\frac{\lambda_N^2 F_{\mu\nu}}{4M_N(p^2-M_N^2)^2}$):

$$g_{N1}^{(1)}(p) = 2i\cos\theta_N M_N^2 F_1^N + i\cos\theta_N(p^2+M_N^2)F_2^N - \sin\theta_N(p^2-M_N^2)F_3^N \qquad (52)$$
$$g_{N2}^{(1)}(p) = 2i\sin\theta_N M_N^2 F_1^N + i\sin\theta_N(p^2+M_N^2)F_2^N + \cos\theta_N(p^2-M_N^2)F_3^N \qquad (53)$$
$$g_{N3}^{(1)}(p) = 2M_N(F_1^N + F_2^N) \qquad (54)$$
$$g_{N4}^{(1)}(p) = 2M_N F_3^N \qquad (55)$$
$$g_{N5}^{(1)}(p) = (-2)\left[\cos\theta_N F_2^N + \sin\theta_N F_3^N \right] = (-2)\,F_N\,\cos(\theta_N-\alpha_N) \qquad (56)$$
$$g_{N6}^{(1)}(p) = (-2)\left[\sin\theta_N F_2^N + \cos\theta_N F_3^N \right] = (-2)\,F_N\,\sin(\theta_N-\alpha_N) \qquad (57)$$

(b) ground state transitions to excited states

As in the case of excited state contributions to the nucleon propagator, we need to sum over all ground state to excited states transitions, and parameterize the spectral function with a few constants. This can be achieved if we take the following expression:

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[8]The transitions in a constant EM background are possible because we are looking at highly off-shell states.

$$\langle \, \Omega \mid \eta_N(0) \mid N \, \rangle_{\theta_q,\theta_G,F_{\mu\nu}\neq 0} - \langle \, \Omega \mid \eta_N(0) \mid N \, \rangle_{\theta_q,\theta_G,F_{\mu\nu}=0}$$

$$\equiv \int d^{2\omega}z \; \langle \, \Omega \mid T \, (\mathcal{L}_{int}(z) \, \eta_N(0)) \mid N(\vec{p}, s_N) \, \rangle_{\theta_q,\theta_G} - \text{ pole term} \qquad (58)$$

$$\equiv e F_{\mu\nu} \left[\, E_N^A \, e^{i\frac{\varphi_N^A}{2}\gamma_5} \, \sigma^{\mu\nu} \; + \; \frac{E_N^B}{M_N} \, e^{i\frac{\varphi_N^B}{2}\gamma_5} \, (\, p^\mu \gamma^\nu - p^\nu \gamma^\mu \,) \right] u(\, \vec{p}, s_N \,) \qquad (59)$$

This is the most general form for a nucleon spinor in the presence of an external EM field; we use the Dirac equation for the (on shell) nucleon spinor $u(\vec{p}, s_N)$ to reduce the invariant tensor structure with four unknown model parameters: $E_N^A, \varphi_N^A, E_N^B, \varphi_N^B$. All of these parameters are invariant functions of $Q^2 \equiv -p^2$. The independent tensors are chosen such that E_N^A, E_N^B are invariant under an $U_A(1)$ rotation, and the phases φ_N^A, φ_N^B transform covariantly. As a crude approximation, we shall neglect the Q^2 dependence of these model parameters and treat them as constants in our calculation[9].

The contributions to the polarization tensor of transitions from the nucleon to excited states is:

$$\sum_{N'} e \, \Pi_N^{\mu\nu}(p)(N \to N') \, F_{\mu\nu} \equiv \int d^{2\omega}x \; e^{ipx}$$

$$\theta(x_0) \, [\, \langle\Omega|\eta_N(x)|N\rangle_{\bar{\theta},F_{\mu\nu}\neq 0} \; - \; \langle\Omega|\eta_N(x)|N\rangle_{\bar{\theta},F_{\mu\nu}=0} \,] \, \langle N|\bar{\eta}_N(0)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}=0} \, +$$

$$+\theta(x_0) \, \langle\Omega|\eta_N(x)|N\rangle_{\bar{\theta},F_{\mu\nu}=0} \, [\, \langle N|\bar{\eta}_N(0)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}\neq 0} \; - \; \langle N|\bar{\eta}_N(0)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}=0} \,] \; -$$

$$-\theta(-x_0) \, [\, \langle\Omega|\bar{\eta}_N(0)|\bar{N}\rangle_{\bar{\theta},F_{\mu\nu}\neq 0} \; - \; \langle\Omega|\bar{\eta}_N(0)|\bar{N}\rangle_{\bar{\theta},F_{\mu\nu}=0}] \, \langle\bar{N}|\eta_N(x)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}=0} \; -$$

$$-\theta(-x_0) \, \langle\Omega|\bar{\eta}_N(0)|\bar{N}\rangle_{\bar{\theta},F_{\mu\nu}=0} \, [\, \langle\bar{N}|\eta_N(x)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}\neq 0} \; - \; \langle\bar{N}|\eta_N(x)|\Omega\rangle_{\bar{\theta},F_{\mu\nu}=0} \,] \quad (60)$$

After collecting terms for each independent tensor basis of the polarization tensor, we obtain (factoring out an overall constant tensor $\frac{e\lambda_N^2 F_{\mu\nu}}{4(p^2 - M_N^2)}$)

$$g_{N1}^{(2)}(p) = 2E_N^A M_N \cos(\frac{\varphi_N^A}{2} + \frac{\theta_N}{2}) \qquad (61)$$

$$g_{N2}^{(2)}(p) = 2E_N^A M_N \sin(\frac{\varphi_N^A}{2} + \frac{\theta_N}{2}) \qquad (62)$$

$$g_{N3}^{(2)}(p) = -2iE_N^A \cos(\frac{\varphi_N^A}{2} - \frac{\theta_N}{2}) \qquad (63)$$

$$g_{N4}^{(2)}(p) = 2i \, [\, E_N^A \sin(\frac{\varphi_N^A}{2} - \frac{\theta_N}{2}) - E_N^B \sin(\frac{\varphi_N^B}{2} - \frac{\theta_N}{2}) \,] \qquad (64)$$

$$g_{N5}^{(2)}(p) = 2i \, \frac{E_N^B}{M_N} \cos(\frac{\varphi_N^B}{2} + \frac{\theta_N}{2}) \qquad (65)$$

$$g_{N6}^{(2)}(p) = 2i \, \frac{E_N^B}{M_N} \sin(\frac{\varphi_N^B}{2} + \frac{\theta_N}{2}) \qquad (66)$$

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[9]A more detailed parameterization including the nucleon pion continuum, which is of importance in terms of both chiral symmetry [18] and quantitative error analysis [19], can be used to check the validity of this approximation [20].

(c) transitions among excited states

We shall apply the duality ansatz to identify the contributions to the polarization tensor of transitions among excited states; the leading contributions from the quark-gluon representation, starting from the continuum threshold s_0^N, are given by

$$Re \ g_{Ni}^{(3)}(p^2) \ (\text{ continuum }) = \frac{1}{\pi} \ Pr. \int_{s_0^N}^{\infty} \frac{Im \ g_{Ni}^{(3)}(s) \ (\text{ quark-gluon })}{p^2 - s} \ ds \qquad (67)$$

The procedure is similar to the case of the nucleon propagator. The quark-gluon calculation will be discussed in the next section.

With all these ingredients, the hadronic representation of the polarization tensor $\Pi_N^{\mu\nu}(p)$ is given by the sum of Eqs. (52) $\sim$ (57), (61) $\sim$ (66), and (67), where we have two nucleon variables, F_2^N, F_3^N (or equivalently, F_N, α_N) and four excited state unknowns, $E_N^A, \varphi_N^A, E_N^B, \varphi_N^B$ which will appear in the phenomenological side of the QCD sum rules.

IV. STUDY OF THE NUCLEON CORRELATION FUNCTION (NCF) FROM THE QUARK-GLUON DEGREES OF FREEDOM

1. Basics

To represent the NCF in terms of the quark–gluon parameters, we use the method of operator product expansion (OPE) to calculate the NCF in QCD [21]. Because the NCF is the vacuum expectation value of composite quark operators, the expansion series of the NCF is a short distance expansion in the coordinate space, or, a Λ_{QCD}^2/Q^2 expansion in momentum space[10]. We truncate the OPE series at dimension six, and the Wilson coefficients are calculated, using diagrammatic rules, in the first order of quark mass m_q and strong coupling constant g_s.

A new feature associated with our problem is the presence of CP violating operators in the OPE series of the NCF. For each CP conserving operator, there exists a parity partner which coming from a γ_5 insertion or a dual transformation, e.g., $\langle \bar{q}q \rangle_{\theta_q, \theta_G}$ vs. $\langle \bar{q}\gamma_5 q \rangle_{\theta_q, \theta_G}$ and $\langle G^2 \rangle_{\theta_q, \theta_G}$ vs. $\langle G\tilde{G} \rangle_{\theta_q, \theta_G}$. This is also true in the case of induced condensates, e.g., $\langle \bar{q}G_{\mu\nu}q \rangle_{\theta_q, \theta_G}$ vs. $\langle \bar{q}G_{\mu\nu}\gamma_5 q \rangle_{\theta_q, \theta_G}$. These CP violating condensates introduce new unknowns in the microscopic representation of the NCF. Therefore, it is important that we know how to evaluate these new condensates such that our calculation has predictive power.

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[10]This is in contrast to the use of OPE in the deep inelastic scattering (DIS) of lepton off nucleon target. Where because of the kinematic (Bjorken limit) and the state of interest (nucleon with a four–momentum p_μ), the expansion series of the correlator in the DIS calculation is a light–cone expansion.

We can classify the CP violating condensates into two classes: the first class consists of $U_A(1)$ chiral covariant operators, e.g., $i\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}$, whose values depend on the representation of the QCD Lagrangian; the second class consists of $U_A(1)$ chiral invariant operators, e.g., $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$ and $\langle\bar{q}\gamma_\mu q\bar{q}\gamma^\mu\gamma_5 q\rangle_{\theta_q,\theta_G}$. The classification is useful because there is a general relation which connects the parity partners in the first class condensates. In particular, one can prove the generalized chiral circle theorem:

Theorem 2

$\Big[\,\langle\bar{q}\Gamma f(G_{\mu\nu})q\rangle_{\theta_q,\theta_G}\,\Big]^2 + \Big[\,i\langle\bar{q}\Gamma f(G_{\mu\nu})\gamma_5 q\rangle_{\theta_q,\theta_G}\,\Big]^2$ *is invariant under $U_A(1)$ chiral rotations*

Compared with the simplest case in Theorem 1, here Γ is an arbitrary Dirac matrix and $f(G_{\mu\nu})$ stands for any gauge invariant function of the gluon field tensor.

Use of the polar representation for quark condensates, see Eqs.(36),(37) allows the combination of $\langle\bar{q}q\rangle_{\theta_q,\theta_G} + \langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\gamma_5$ to be written as

$$\langle\bar{q}q\rangle_{\theta_q,\theta_G} + \langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G}\gamma_5 = -R_q e^{-i\theta_G\gamma_5} \tag{68}$$

The phase convention for θ_G is chosen such that $\langle\bar{q}q\rangle_{\theta_q,\theta_G}$ is negative for $\theta_q = \theta_G = 0$.

The generalized chiral circle theorem can be applied to induced condensates of parity partners, e.g.:

$$g_s\bar{q}G_{\mu\nu}q, \qquad\qquad\qquad g_s\bar{q}\gamma_5 G_{\mu\nu}q; \tag{69}$$
$$g_s\bar{q}\sigma_{\mu\alpha}G^\alpha_\nu q - (\mu\leftrightarrow\nu), \qquad g_s\bar{q}\gamma_5\sigma_{\mu\alpha}G^\alpha_\nu q - (\mu\leftrightarrow\nu). \tag{70}$$

The susceptibility constants associated with these induced condensates are defined as follows:

$$g_s\langle\bar{q}G_{\mu\nu}q\rangle_{\theta_q,\theta_G} \equiv \kappa_q F_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} - i\tilde{\kappa}_q\tilde{F}_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{71}$$
$$g_s\langle\bar{q}\gamma_5 G_{\mu\nu}q\rangle_{\theta_q,\theta_G} \equiv -i\xi_q\tilde{F}_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} + \tilde{\xi}_q F_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{72}$$
$$g_s\bar{q}\sigma_{\mu\alpha}G^\alpha_\nu q - (\mu\leftrightarrow\nu) \equiv \eta_q F_{\mu\nu}\langle\bar{q}q\rangle_{\theta_q,\theta_G} - i\tilde{\eta}_q\tilde{F}_{\mu\nu}\langle\bar{q}\gamma_5 q\rangle_{\theta_q,\theta_G} \tag{73}$$

Using the generalized chiral circle theorem, we have

$$\tilde{\kappa}_q = \xi_q, \qquad \tilde{\xi}_q = \kappa_q, \qquad \tilde{\eta}_q = \eta_q. \tag{74}$$

If we assume that all the susceptibility constants are proportional to the quark charge e_q times a flavor independent constant, then we can rewrite the susceptibility constants

$$\kappa_q = e_q\kappa, \qquad \xi_q = e_q\xi, \qquad \eta_q = e_q\eta. \tag{75}$$

As to the second class condensates, we are only able to use the anomalous Ward identity to obtain the value of $\langle G\tilde{G}\rangle_{\theta_q,\theta_G}$, which is the lowest dimensional chirally invariant

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condensate in our calculation[11]. There are many unknown dimension six chirally invariant condensates, e.g., $\langle \bar{q}\gamma_\mu q \bar{q}\gamma^\mu \gamma_5 q \rangle_{\theta_q,\theta_G}$, which cannot be related to their parity partners using the chiral circle theorem, and we shall follow the general practice to factorize these four-quark condensates into products of dimension three quark condensates.

Having explained the subtlety of the calculations of NCF in QCD, we can present the result in terms of various coefficients of the invariant tensors[12]:

2. **Quark-Gluon Representation of the Nucleon Propagator $\Pi_N^{(0)}(p)$**

The $f_1^p(p^2) \, \hat{p}$ sum rule receives the contributions shown in Fig.1 .

Fig.1 (a) represents the contribution of the operator I; Fig.1 (b) represents the contribution of the operator $m_q R_q \cos\bar{\theta}_q$; Fig.1 (c) represents the contribution of the operator $R_q^2 \approx \langle \bar{q}\gamma_\mu q \, \bar{q}\gamma^\mu q \rangle_{\theta_q,\theta_G}$.

The Feynman diagrams contribute to $f_2^p(p^2)e^{i\phi^p(p^2)\gamma_5}$ sum rule are given in Fig.2.

Fig.2 (a) represents the contribution of the operator $m_q e^{i\theta_q\gamma_5}$; Fig.2 (b) represents the contribution of the operator $R_q e^{i\theta_G\gamma_5}$;

3. **Quark-Gluon Representation of the Polarization Tensor $\Pi_{\mu\nu}^N(p)$**

Out of the six basis tensors in the polarization tensor, only three of them are useful. This is because the $\sigma^{\mu\nu}(I,\gamma_5)$ and the $(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5$ sum rules contain infrared singularities which indicate operator mixings with induced condensates whose values are unknown[13] [10]. Consequently, we only list three sum rules below:

The $\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta \cdot \gamma_5$ sum rule contains the nucleon magnetic moments and the OPE diagrams are shown in Fig.3.

Fig.3 (a) represents the contribution of the operator $F_{\mu\nu}$; Fig.3 (b) represents the contribution of the operator $m_q R_q \cos\bar{\theta}_q F_{\mu\nu}$; Fig.3 (c) represents the contribution of the operator R_q^2.

The $\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)$ sum rule, being a chirally covariant tensor, contains both the anomalous magnetic moment F_2^N and the electric dipole moment F_3^N, or equivalently, F_N and α_N. The diagrams in the OPE series are shown in Fig.4.

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[11]Taking the vacuum expectation value of the anomalous Ward identity, we obtain $\langle G\tilde{G}\rangle_{\theta_q,\theta_G} \propto m_q R_q \sin\bar{\theta}_q$. See [22].

[12]We do not list the complete diagrams which contribute to the OPE series of the NCFs. Instead, only terms which survive after further simplifications (see discussions in the next section) are retained in this section.

[13]For example, the tensor operators $m_q F_{\mu\nu}$, $m_q \tilde{F}_{\mu\nu}$ and $\bar{q}\sigma_{\mu\nu}q$, which appear in the $\sigma^{\mu\nu}(I,\gamma_5)$ sum rule, and the operators $F_{\mu\nu}G\tilde{G}$ and $\bar{q}\gamma_\mu q\bar{q}\gamma_\nu\gamma_5 q$ in the $(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5$ sum rule, are mixed under renormalization group.

Fig.4 (a) represents the contribution of the operator $m_q e^{i\theta_q\gamma_5}F_{\mu\nu}$, Fig.4 (b) represents the contribution of the operator $R_q e^{i\theta_G\gamma_5}F_{\mu\nu}$, Fig.4 (c) represents the contribution of the operator $\langle\bar{q}\sigma_{\mu\nu}q\rangle_{\theta_q,\theta_G}$.

V. ANALYSIS OF QCD SUM RULES (QSR) FOR THE NUCLEON CORRELATION FUNCTION: PART ONE

The QCD sum rules (QSR) for the NCFs in the presence of an external electromagnetic field will be analyzed in this section. The main emphasis will be on

- the consistency of the QCD sum rules for NEDMs with symmetry constraints on strong CP violations [9], and

- an order-of-magnitude estimate of hadronic variables (e.g., nucleon masses M_N, residues λ_N, EM moments μ_N, d_N) extracted from the QCD sum rules.

This is a primary analysis in the sense that we shall treat the approximate sum rules (due to truncations of the OPE series and the hadronic parameterizations of the spectral functions) as identities, and solve for the hadronic observables as unknown variables in the sum rule equations. A more complete study will be presented in the next section.

1. **QCD sum rules of the NCFs in the momentum space**

   Here we summarize the results of our calculations in the last two sections by listing all the invariant coefficient functions of various independent basis tensors[14] for the proton correlation function in momentum space. The equalities relating hadronic parameterizations and the OPE calculations for various invariant coefficient functions are referred to as QCD sum rules. It should be kept in mind that these two representations of NCFs are derived from different expansions and truncations (a hadronic complete set in the former and $1/Q^2$ in the latter) of the same correlation functions. Therefore, these equalities are at best approximate identities whose validity are empirical.

   The QCD sum rules for the proton propagator $\Pi_p^{(0)}$ representation are given by

$$\Pi_p^{(0)} = f_1^p(p^2)\hat{p} + f_2^p(p^2)e^{i\phi^p(p^2)\gamma_5} \tag{76}$$

(a) $f_1^p(p^2)$ ($\hat{p}$ sum rule)

$$\frac{\lambda_p^2}{p^2 - M_p^2} + \text{ continuum } = \frac{p^4\ln(-p^2)}{4(2\pi)^4} + \frac{4m_u a_u \cos\bar{\theta}_u \ln(-p^2)}{(2\pi)^4} - \frac{2a_u^2}{3(2\pi)^4 p^2} \tag{77}$$

(b) $f_2^p(p^2)e^{i\phi^p(p^2)\gamma_5}$ (I, γ_5 sum rule)

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[14]We do not list the $\sigma_{\mu\nu}(I,\gamma_5)$ sum rules because of the infrared problem [10].

$$\frac{\lambda_p^2 M_p}{p^2 - M_p^2} e^{i\theta_p \gamma_5} + \text{continuum} = \frac{m_d p^4 \ln(-p^2)}{4(2\pi)^3} e^{i\theta_d \gamma_5} + \frac{a_d p^2 \ln(-p^2)}{(2\pi)^4} e^{i\theta_{Gd}\gamma_5} \tag{78}$$

The QCD sum rules for the proton polarization tensor $\Pi_{\mu\nu}^p(p)$ in the polar form representation are given by

$$\Pi_{\mu\nu}^p(p) = \sum_{i=1}^{i=6} g_i^p(p^2) \cdot T^i \tag{79}$$

(c) $g_3^p(p^2)$ ($\epsilon^{\mu\nu\alpha\beta} p_\alpha \gamma_\beta \cdot \gamma_5$ sum rule)

$$\frac{2\lambda_p^2(F_1^p + F_2^p)}{4(p^2 - M_p^2)^2} + \frac{2\lambda_p^2 E_A^p \cos(\frac{\varphi_A^p}{2} - \frac{\theta_p}{2})}{4(p^2 - M_p^2)} + \text{continuum}$$

$$= e_d \left[\frac{p^2 \ln(-p^2)}{2(2\pi)^4} + \frac{m_u a_u \cos \bar{\theta}_u}{p^2} + \frac{a_u^2}{3p^4} \right] + (\propto e_u) \tag{80}$$

(d) $g_4^p(p^2)$ ($(p^\mu \gamma^\nu - p^\nu \gamma^\mu) \cdot \gamma_5$ sum rule)

$$\frac{2\lambda_p^2 F_3^p}{4(p^2 - M_p^2)^2} + \frac{2\lambda_p^2 \left[E_A^p \sin(\frac{\varphi_A^p}{2} - \frac{\theta_p}{2}) - E_B^p \sin(\frac{\varphi_B^p}{2} - \frac{\theta_p}{2}) \right]}{4(p^2 - M_p^2)} + \text{continuum}$$

$$= (\propto m_q R_q \sin \bar{\theta}_q) \tag{81}$$

(e) $g_5^p(p^2) + i g_6^p(p^2) \gamma_5$ ($\hat{p}(p^\mu \gamma^\nu - p^\nu \gamma^\mu)$ sum rule)

$$\frac{(-2)\lambda_p^2 F_p}{4 M_p (p^2 - M_p^2)^2} e^{i(\theta_p + \alpha_p)\gamma_5} + \frac{2\lambda_p^2 E_B^p}{4 M_p (p^2 - M_p^2)} e^{i(\frac{\varphi_B^p + \theta_p}{2})\gamma_5} + \text{continuum}$$

$$= e_u \left[\frac{m_d \ln(-p^2)}{(2\pi)^4} \right] e^{i\theta_d \gamma_5} + \left[e_d \frac{\chi a_d \ln(-p^2)}{3(2\pi)^4} + e_u \frac{a_d}{(2\pi)^4 p^2} \right] e^{i\theta_{Gd}\gamma_5} \tag{82}$$

Here we define $a_q \equiv (2\pi)^2 R_q \approx 0.55(GeV)^3$.

The neutron sum rules can be obtained from the proton ones by doing an isospin rotation, namely, replacing the hadronic observables, e.g., M_p by M_n, λ_p by λ_n; and the QCD parameters, e.g., m_d by m_u, and R_d by R_u etc.

2. The use of the Borel transform

In principle, the QCD sum rules, as summarized above, can be used to extract information on hadronic variables in terms of QCD parameters. However, the validity of the matching between these two representations for the NCFs is severely limited by the convergence property of the OPE series and the uncertainty of the higher state contributions. Fortunately, there are several prescriptions to improve the sum rule relations. Specifically, we wish to generate a set of improved sum rules such that the OPE series have better convergence and the contributions from a hadronic parametrization

is dominated by the ground state observables. For this purpose, we use the Borel transformation, which is defined as [16]:

$$\mathcal{B}_{M_B}[f(Q^2)] \equiv \lim_{\substack{Q^2 \to \infty \\ n \to \infty \\ M_B \equiv \frac{Q^2}{n} fixed}} \left(\frac{1}{n!}\right)(Q^2)^{n+1}\left(-\frac{d}{dQ^2}\right)^{n+1} \tag{83}$$

We need to apply this transformation[15] on both sides of the sum rule relations:

For instance, on the OPE side of the QCD sum rules, we have

$$\mathcal{B}_{M_B}[(Q^2)^m \ln(Q^2)] = (-1)^{m+1} m! (M_B^2)^m \tag{84}$$

$$\mathcal{B}_{M_B}\left[\left(\frac{1}{Q^2}\right)^k\right] = \frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^k \tag{85}$$

Thus, after Borel transformation, the higher dimensional operators in the OPE series receive further suppression with factorial factors.

On the phenomenological side of the QCD sum rules, we have

$$\mathcal{B}_{M_B}\left[\left(\frac{1}{Q^2+M^2}\right)^k\right] = \frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^k e^{-\frac{M^2}{M_B^2}} \tag{86}$$

The power suppressions of the excited state contributions are Borel transformed into exponential ones.

Having established the usefulness of the Borel transformation, we need to specify how the matching of sum rule relations can be realized. As we have mentioned before, the identities we have derived cannot be exact for all values of M_B^2. A choice of "matching region" has to be made; such a choice is a compromise between different convergence properties of the two representations of the NCFs. On the OPE side, we prefer a large value of M_B to suppress power corrections (see Eqs.(84), (85)); on the phenomenological side, we prefer a small value of M_B to enhance ground state observables (see Eq.(86)). In view of this, the matching of the sum rule relations only works for a finite range of the Borel mass M_B, and the region of matching the QCD calculations and hadronic parametrizations is generally referred to as a Borel window.

It is an empirical fact that within the Borel window, physical quantities have a mild dependence on the squared Borel mass M_B^2. Hence, in the primary analysis, we can choose a value for the squared Borel mass M_B^2 close to the ground state mass M_N^2 to obtained a rough estimation for the observables of interest.

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[15]Notice that, after the Borel transformation, the virtual 4–momentum variable $Q^2 \equiv -p^2$ appearing in the momentum space sum rules is replaced by the squared Borel mass M_B^2.

A technical remark regarding the continuum contribution:

The continuum contributions to both nucleon propagator and the polarization tensor are given by the duality ansatz Eq.(46), (67). After applying the Borel transform Eq.(86) (with $k = 1$) on both sides of Eqs.(46), (67), we get

$$\mathcal{B}_{M_B}\left[Re\ f_i^N(p^2)(\text{ Continuum })\right] = -\frac{1}{\pi M_B^2} Pr \int_{s_0^N}^{\infty} \left[Im\ f_i^N(s)(\text{ quark-gluon })\right] e^{-\frac{s}{M_B^2}} ds$$

(87)

The leading OPE terms of the NCF are of the form of $(p^2)^n \ln(-p^2 - i\epsilon)$, which has an imaginary part equal to $-\pi(p^2)^n$. Thus the right hand side of the Eq.(87) is a truncated Laplace transformation of a polynomial in p^2. Since the continuum contributions share a similar form as the leading OPE terms on the quark–gluon side of the NCF, we can combine these two terms together, and define the following functions:

$$E_n(s, w) \equiv 1 - \int_w^{\infty} dt\ e^{-\frac{t}{s}}\ t^n \tag{88}$$

$$E_0(s, w) = 1 - e^{-\frac{w}{s}} \tag{89}$$

$$E_1(s, w) = 1 - e^{-\frac{w}{s}}(\frac{w}{s} + 1) \tag{90}$$

$$E_2(s, w) = 1 - e^{-\frac{w}{s}}(\frac{w^2}{2s^2} + \frac{w}{s} + 1) \tag{91}$$

All functions E_n act as a high–energy cutoff for the leading OPE series, with $E_n(s, w) \approx 0$ if $s \geq w$. Since the continuum threshold s_0^N represent an average parameter for the excited state spectrum, we shall choose $w = s_0^N = 1.75 GeV^2$ and write $E_n(M_B^2) \equiv E_n(s = M_B^2, w = 1.75 GeV^2)$.

3. **Borel transform–improved sum rules**

The Borel transform–improved sum rules for the proton propagator $\Pi_0^p(p)$ is given by

(a) $f_1^p(M_B^2)$

$$M_B^6 E_2(M_B^2) + 4m_u a_u \cos\bar{\theta}_u M_B^2 E_0(M_B^2) + \frac{4a_u^2}{3} = \tilde{\lambda}_p^2 e^{-\frac{M_p^2}{M_B^2}} \tag{92}$$

(b) $f_2^p(M_B^2)e^{i\phi^p(p^2)\gamma_5}$

$$2M_B^6 E_2(M_B^2)m_d e^{i\theta_d\gamma_5} + 2M_B^4 E_1(M_B^2)a_u e^{i\theta_{Gd}\gamma_5} = \tilde{\lambda}_p^2 M_p e^{-\frac{M_p^2}{M_B^2}} e^{i\theta_p\gamma_5} \tag{93}$$

where $\tilde{\lambda}_N^2 \equiv 2(2\pi)^4 \lambda_N^2$. For the proton polarization tensor $\Pi_{\mu\nu}^p(p)$, we obtain the following sum rules:

(c) $g_3^p(M_B^2)$

$$e_u M_B^4 E_1(M_B^2) + e_d m_u a_u \cos\bar{\theta}_d + \frac{a_u^2}{3M_B^2}[-(e_d + \frac{2e_u}{3}) + (\propto e_u)]$$

$$= \frac{\tilde{\lambda}_p^2}{4} e^{-\frac{M_p^2}{M_B^2}} \left[\frac{F_1^p + F_2^p}{M_B^2} + E_p^A \cos(\frac{\varphi_p^A}{2} - \frac{\theta_p}{2}) \right] \tag{94}$$

(e) $g_5^p(p^2) + ig_6^p(p^2)\gamma_5$

$$[\, 4e_u a_d + (\propto e_d)\,] e^{i\theta_{Gd}\gamma_5} - [\, 4e_u m_d + (\propto e_d)\,] M_B^2 e^{i\theta_d\gamma_5}$$

$$= \frac{\tilde{\lambda}_p^2}{M_p} e^{-\frac{M_N^2}{M_B^2}} \left[\frac{F_p}{M_B^2} e^{i(\theta_p + \alpha_p)\gamma_5} + E_p^B e^{i(\frac{\varphi_p^B + \theta_p}{2})\gamma_5} \right] \tag{95}$$

4. Manipulation of the QCD Sum Rules

While we have identified the hadronic unknowns and written down the sum rule equations to relate hadronic observables in terms of quark–gluon parameters, we need to take some further simplifications to make the solutions of these complicated relations manageable[16].

The three steps we adopt to simplify the sum rule relations are discussed in order:

(a) The use of isospin symmetry

It is important that the symmetry constraints on the strong CP violations require that all quarks have finite masses. In particular, the existence of any massless quark leads to no strong CP violation. In such a limit, the flavor symmetry is broken and we have to make the flavor dependence in our calculation explicit. While it is possible to realize these limits in the QCD sum rule approach [22], we opt for isospin symmetry to simplify our calculations. Consequently, for the quark-gluon parameters, we take $m_u = m_d = m_q$ and $R_u = R_d = R_q$. It can be shown that in this case, θ_u can be chosen to be equal to θ_d and, as a consequence of Crewther's condition, $\theta_{Gu} = \theta_{Gd} = \theta_G/2$. See Eqs.(38) (39).

On the other hand, since we do not include QED corrections, which break the isospin symmetry, in the calculations of the hadronic masses and residues, we can take $M_p = M_n$ and $\lambda_p = \lambda_n$. Thus, the use of isospin symmetry greatly reduces the unknowns and the input parameters in the sum rule relations.

The other advantage of this simplification, as pointed out by B.L. Ioffe and A.V. Smilga [10], is that we can eliminate the induced condensates which generate the unknown susceptibility constants from the proton and neutron sum rules by taking certain combinations with different quark charge dependences.

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[16]However, we emphasize that none of these simplifications are absolutely necessary for the physical content of the further discussions. The choice of these simplifications should be regarded as pedagogic and to help us isolate issues of strong CP violation from other complications in the strong interactions.

For example, in the $\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)(1, \gamma_5)$ sum rule we can multiply the proton sum rule Eq.(82) by e_u and subtract the corresponding neutron sum rule multiplied by e_d. This eliminates the contributions of the induced condensates $\langle \bar{q}\sigma_{\mu\nu}q \rangle_{\theta_q,\theta_G} \propto \chi \langle \bar{q}q \rangle_{\theta_q,\theta_G}$. For the $\epsilon^{\mu\nu\alpha\beta}p_\alpha p_\beta \cdot \gamma_5$ sum rule Eq.(80), we repeat the similar procedure, but with e_d times the proton sum rule minus e_u times the neutron one. As a bonus, we also eliminate the infrared singularity coming from the operator $m_q F_{\mu\nu}$.

(b) The elimination of excited state parameters in the polarization tensor

We shall not concern ourselves with the details of the excited states in this calculation. With the assumption that the ground state to excited state transition can be approximated by momentum independent constants Eq.59, one can observe that such a contribution has a different Borel mass dependence, as compared to the ground state observables, see Eqs.(94) (95). We can apply the differential operator

$$ 1 - M_B^2 \frac{\partial}{\partial M_B^2} \tag{96} $$

on both sides of the sum rule equations Eqs.(94), (95) to eliminate the contributions of ground to excited state transitions in the sum rule relations.

(c) The problem of operator mixing in the $(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5$ sum rule

After all these simplifications, we have seven unknowns $\lambda_N, M_N, \theta_N, F_p, \alpha_p, F_n, \alpha_n$ in the sum rules relations. While the nucleon propagator $\Pi_N^{(0)}$ gives three independent sum rules ($\hat{p}, 1, \gamma_5$) for the nucleon variables λ_N, M_N, θ_N, we have only three useful identities Eqs.(94),(95) in the polarization tensor $\Pi_{\mu\nu}^N$ for four EM moments $F_p, \alpha_p, F_n, \alpha_n$! The leading OPE contribution to the tensor $(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5$ is proportional to the operator $F_{\mu\nu}\langle G\tilde{G} \rangle_{\theta_q,\theta_G}$, whose Wilson coefficient has a $\ln m_q$ infrared singularity[17]. One can argue that such an infrared singularity should be included in the definition of QCD condensates, which requires a separation scale instead of a small quark mass. The difficulty here is to show our calculation is independent of the choice of a separation scale, as required by the renormalization group analysis. The situation is further complicated by the mixing of operators under renormalization. Without a detailed analysis of the operator mixing problem, we can not obtain useful information from this tensor. Consequently, we need to introduce a phenomenological parameter β for the ratio of F_3^p to F_3^n. By comparing the Feynman diagrams contributing to the tensor $(p^\mu\gamma^\nu - p^\nu\gamma^\mu) \cdot \gamma_5$ for both proton and neutron, we find the only difference is the charge dependence of u and d quarks, the quark masses and chiral radii do not enter this sum rule because of the chiral property of this sum rule. The charge dependence seems to indicate that $\beta = e_u/e_d$. Henceforth, we shall assume such a relation in our subsequent discussion.

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[17]We need to calculate a quark propagator in the presence of two or three constant external fields ($G_{\mu\nu}$ and/or $F_{\mu\nu}$), which is infrared divergent if the quark mass is zero.

5. **Symmetry Constraints from the QCD Sum Rule Relations**

Despite the complicated appearance of the sum rule relations, it is possible to show that, due to the use of the polar form in both hadronic and quark–gluon representations of the NCF, our sum rule calculations satisfy the symmetry constraints on the strong CP violation [9] without an explicit solution of the CP violating observable, e.g.,NEDM.

To see this, we focus on the $(1, \gamma_5)$ sum rules, Eq.(93) in the nucleon propagator and the $\hat{p}(p^\mu \gamma^\nu - p^\nu \gamma^\mu)(1, \gamma_5)$ sum rules, Eq.(95) in the polarization tensor. Both sets of sum rules are associated with chirally covariant tensors and receive contributions from chirally covariant condensates. In a shorthand notation, we can rewrite the sum rules in the following forms:

$$A \; m_q \; e^{i\theta_q \gamma_5} + B \; R_q \; e^{i\theta_{Gq} \gamma_5} = C e^{i\theta_N \gamma_5} \tag{97}$$
$$A' \; m_q \; e^{i\theta_q \gamma_5} + B' \; R_q \; e^{i\theta_{Gq} \gamma_5} = C' e^{i(\theta_N + \alpha_N)\gamma_5} \tag{98}$$

We emphasize that the structure of the sum rules, as summarized above, is independent of the approximation scheme which only affects the values of the numerical coefficients A, B, A' and B'. Specifically, in terms of the Wilson coefficients appearing in the (Borel transformed) QCD sum rules, these numerical coefficients are

$$A = 2M_B^6 E_2(M_B^2) \tag{99}$$
$$B = 2M_B^4 E_1(M_B^2) \tag{100}$$
$$A' = 4(e_d^2 - e_u^2)M_B^4 E_0(M_B^2) \tag{101}$$
$$B' = -4(e_d^2 - e_u^2)M_B^2 \tag{102}$$

In the two sets of sum rules Eqs.(97) (98), we have four hadronic unknowns C, C', θ_N, and α_N to be determined. The relation with the nucleon variables is that C is a function of λ_N and M_N, C' is a function of F_p and F_n, with an assumption[18] that

$$\tan^{-1} \frac{F_p^3}{F_p^2} = \tan^{-1} \frac{F_n^3}{F_n^2} \equiv \alpha_N \tag{103}$$

we rearrange the seven nucleon variables into four unknowns.

The former sum rule, Eq.(97) contains the nucleon chiral phase θ_N, the latter Eq.(98) has both θ_N and α_N, in the hadronic representations of the NCF. It is the relative (chiral) phase difference that defines the physically measurable (and chirally invariant) dimensionless ratio d_n/μ_n^a, which determines the violation of the CP symmetry.

On the other hand, the OPE series for both sum rules are organized in such a way that the explicit chiral symmetry breaking parameter m_q and the spontaneous chiral

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[18]This assumption is chosen for its simplicity and is not equivalent to the conjectured value for $\beta = e_u/e_d$.

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symmetry breaking parameter R_q are on equal footings[19]. Such a representation of the NCF is particularly useful for examining the symmetry constraints on the strong CP violation. Namely, in either limit $m_q \to 0$ or $R_q \to 0$, the nucleon chiral phase θ_N becomes θ_{Gq} or θ_q, respectively, and the relative phase $\alpha_N \equiv \tan^{-1}(d_n/\mu_n^a)$ has to vanish. Thus, there is no strong CP violation if chiral symmetry is exact in QCD and CP violating observables must be proportional to the product of m_q and R_q. Furthermore, the periodic structure in the chiral phases naturally generates a $\sin\bar{\theta}$ factor[20], which combined with the two chiral symmetry breaking parameters m_q and R_q, implies that CP violating observables are proportional to $\langle G\tilde{G}\rangle_{\bar{\theta}}$[21].

We can solve the equations (97), (98) for the hadronic unknowns $C, C', \theta_N, \alpha_N$ as follows:

$$C_N^2 = A^2 m_q^2 + B^2 R_q^2 + 2ABm_q R_q \cos\bar{\theta} \tag{104}$$

$$C'^2_N = A'^2 m_q^2 + B'^2 R_q^2 + 2A'B' m_q R_q \cos\bar{\theta} \tag{105}$$

$$\tan\theta_N = \frac{Am_q \sin\theta_q + BR_q \sin\theta_G}{Am_q \cos\theta_q + BR_q \cos\theta_G} \tag{106}$$

$$\tan\alpha_N = \frac{(AB' - BA')m_q R_q \sin\bar{\theta}}{AA'm_q^2 + BB'R_q^2 + (AB' + BA')m_q R_q \cos\bar{\theta}} \tag{107}$$

The nucleon variables are given by the following expressions:

$$M_N = \frac{C_N}{M^6 E_2(M_B^2) + 4m_q R_q M_B^2 E_0(M_B^2) \cos\bar{\theta}_q + 4R_q^2/3} \tag{108}$$

$$\lambda_N = M_B^6 E_2(M_B^2) + 4m_q R_q M_B^2 \cos\bar{\theta}_q E0(M_B^2) + 4R_q^2/3 \tag{109}$$

$$F_2^p = \frac{M_N^2}{e_u^2 - e_d^2}(e_u U(M_B^2, \bar{\theta}_q) - e_d K(M_B^2, \bar{\theta}_q) + e_d^2) \tag{110}$$

$$F_2^n = \frac{M_N^2}{e_u^2 - e_d^2}(e_d U(M_B^2, \bar{\theta}_q) - e_u K(M_B^2, \bar{\theta}_q) + e_d e_u) \tag{111}$$

$$F_3^p = \frac{M_N^2}{e_u^2 - e_d^2} e_u V(M_B^2, \bar{\theta}_q) \tag{112}$$

$$F_3^n = \frac{M_N^2}{e_u^2 - e_d^2} e_d V(M_B^2, \bar{\theta}_q) \tag{113}$$

where

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[19]The form of the chirally covariant sum rules is a concrete realization of the strong CP torus of QCD, as described in refer. [9].

[20]This is the simplest periodic function, which is regular and approaches zero as $\bar{\theta}$ vanishes.

[21]Taking the expectation value of the anomalous Ward identity for the flavor singlet axial current, one can show that $\langle G\tilde{G}\rangle_{\bar{\theta}}$ is propotional to the product of m_q, R_q and $\sin\bar{\theta}$.

$$K(M_B^2, \bar{\theta}_q) \equiv \left(1 - M_B^2 \frac{\partial}{\partial M_B^2}\right)(e_u^2 - e_d^2)\left[\frac{4R_q^2/3 + 4m_q R_q M_B^2 E_0(M_B^2)\cos\bar{\theta}_q}{M^6 E_2(M_B^2) + 4m_q R_q M_B^2 E_0(M_B^2)\cos\bar{\theta}_q + 4R_q^2/3}\right] \tag{114}$$

$$U(M_B^2, \bar{\theta}_q) \equiv \left(1 - M_B^2 \frac{\partial}{\partial M_B^2}\right)\left[\frac{AA'm_q^2 + (AB' + A'B)m_q R_q \cos\bar{\theta}_q + BB'R_q^2}{C_N^2}\right] \tag{115}$$

$$V(M_B^2, \bar{\theta}_q) \equiv \left(1 - M_B^2 \frac{\partial}{\partial M_B^2}\right)\left[\frac{(AB' - A'B)\langle G\tilde{G}\rangle_{\theta_q,\theta_G}}{2C_N^2}\right] \tag{116}$$

Given the small value of the current quark mass, $m_q \approx 5MeV$, and the relative minus sign in the numerator of Eq.(107), it is possible that the CP violating observables could be dynamically suppressed without a tiny $\bar{\theta}$ parameter. We shall examine such an interesting scenario in the next section.

6. **Order of Magnitude Estimations of the Hadronic Observables from the QCD Sum Rules**

Before we embark on a complete analysis of the sum rule relations, it is useful to estimate the relative sizes of the hadronic observables. The qualitative features of the estimations also provide a test whether the underlying assumptions in the sum rule calculation work for the case in which we are interested. It is an empirical fact that the Borel window lies around the $1GeV$ region and we can choose a value for the Borel mass M_B to be the nucleon mass $\approx 1GeV$, if the Borel mass dependence of the hadronic observables is weak. Other inputs needed for the estimations of the hadronic observables include the current quark mass $m_q \approx 5MeV$, and the quark chiral radius $R_q \approx -\langle \bar{q}q\rangle_{\theta_q=\theta_G=0} = (240MeV)^3$. The latter gives $a_q = 0.55(GeV)^3$.

Substituting these numbers into Eqs.(99) $\sim$ (100), in the small $\bar{\theta}$ limit, the functions Eqs.(108) $\sim$ (113) reduce to

$$M_N(\bar{\theta}) = 0.9GeV \tag{117}$$
$$\tilde{\lambda}^2(\bar{\theta}) = 2.5(GeV)^6 \tag{118}$$
$$F_2^p(\bar{\theta}) = 3.1 \tag{119}$$
$$F_2^n(\bar{\theta}) = -2.1 \tag{120}$$
$$F_3^p(\bar{\theta}) = 10^{-2}\sin\bar{\theta} \tag{121}$$
$$F_3^n(\bar{\theta}) = -0.5 \times 10^{-2}\sin\bar{\theta} \tag{122}$$

Comparing the ratio F_3^N/F_2^N with the current experiment upper bound on the nEDM $d_n/\mu_n^a \leq 10^{-11}$, we obtain an upper bound on the strong CP violating parameter $\bar{\theta} \leq 10^{-9}$.

We emphasize that the upper bound on the $\bar{\theta}$ parameter is obtained from a calculation without assuming a perturbative expansion on the $\bar{\theta}$ parameter. Consequently, we can derive an upper bound on the ratio d_n/μ_n^a with respect to the $\bar{\theta}$ parameter. Such information is an intrinsic property of the CP conserving ($\bar{\theta} = 0$) QCD and provides a dynamical suppression mechanism to the CP violating observables for solving the strong CP problem. Unfortunately, the number we have derived $\max_{0 \leq \bar{\theta} \leq 2\pi} \frac{d_n}{\mu_n^a} \leq$

10^{-2}, is not small enough to achieve this end. Therefore, we conclude that the solution to the strong CP problem has to lie beyond QCD.

VI. ANALYSIS OF QCD SUM RULES (QSR) FOR THE NUCLEON CORRELATION FUNCTION: PART TWO

This section aims at a more careful study of the QSRs for the NCFs in the presence of an external EM field. We shall extract various nucleon variables, e.g., the nucleon masses M_N, nucleon residues λ_N and their EM moments F_2^N, F_3^N, from the QSRs in a more rigorous manner. The emphasis here is to use the QCD sum rule method to obtain quantitive results for the hadronic variables, including an analysis and/or estimate of the errors and uncertainties in our calculations.

We first discuss how to choose a region for the matching between two representations of the NCFs; then we extract the hadronic observables from a least square fit. The possible errors and uncertainty are discussed in the third part and we summarize the results obtained from the sum rule analysis in the last part.

1. Choice of the Borel Window

As we have emphasized before, the hadronic and quark-gluon representations of the NCFs are based on two different expansion schemes and there is no a priori reason that, under our truncation and approximation, these two expansions should provide the same information of the NCFs. While it is possible to enlarge the overlap and improve the convergence with the use of the Borel transform, a matching region between both sides of the sum rules has to be specified. In view of the different convergence properties, we use the following criterion to define the Borel window.

First of all, on the quark gluon representation of the NCFs, we require that the highest dimension condensates contribute to the whole series less than 20%. This leads to a lower bound on the Borel mass squared $M_B^2 \geq 0.8$ GeV^2.

Secondly, for the hadronic parametrization, we restrict the contribution from the continuum to be no greater than 20% of the leading OPE contributions to the NCF. This leads to an upper bound on the Borel mass squared $M_B^2 \leq 1.2$ GeV^2.

Thirdly, the extractions[22] of the hadronic variables in the region specified above introduce additional uncertainty. Presumably, the third uncertainty is not independent from the ones associated with the convergence requirement. However, such a correlation is not transparent in our analysis, and we simply take the intersection region of the three criterions to define the Borel window, which lies between $0.8 GeV^2$ to $1.2 GeV^2$.

2. Extraction of the Hadronic Observables from the QCD Sum Rules

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[22]There is no standard procedure to extract hadronic variables in the sum rules approach. In our case, we average the hadronic variables over the Borel window and use the χ^2 to represent the uncertainty with our extractions.

Given the simplifications discussed in the previous section, we obtain several relations which give the hadronic observables as functions of the squared Borel mass. As we have emphasized, if the matching between the hadronic and quark–gluon representations of the NCF is realized in the given Borel window, all the hadronic observables should have only a mild dependence on the squared Borel mass. Therefore, we plot[23] all six chirally invariant hadronic observables, nucleon mass M_N, nucleon residue $\tilde{\lambda}_N^2$, proton and neutron magnetic moments $1 + F_2^p, F_2^n$ and electric dipole moments F_3^p, F_3^n, as functions of the squared Borel mass M_B^2 in Fig.5 and Fig.6. The final values of these hadronic observables are determined by averaging the functions over the Borel window and the uncertainty is given by the χ^2 of this average. Since it is necessary to have such χ^2 uncertainty to be smaller than the uncertainty associated with the choice of Borel window, we shall consider the result obtained from our sum rule analysis to lie within 20% uncertainty.

3. **Error Analysis**

We list a few comments on the errors and uncertainties related to this calculation.

On the OPE calculations of the NCF:

(1) The convergence of the OPE expansion:

The OPE series for the NCF in momentum space is a Λ_{QCD}^2/Q^2 expansion, where we assume that all vacuum condensates are proportional to Λ_{QCD} to a certain power, with a numerical coefficient of order $O(1)$. While there is no systematic way to estimate the large order behavior of the coefficients and test this assumption, the expansion series could be an asymptotic series and has no convergence radius at all. We have to content ourselves with the naive estimate from the Borel window analysis and estimate the contributions from the higher dimensional condensates to be below 20%.

(2) Approximations in the calculations of the Wilson coefficients:

When we calculate the OPE series of a correlation function, the Wilson coefficients are expanded in the powers of "small" parameters, α_s, m_q^2/Q^2 and we make truncations of the series to obtain an approximate results for the Wilson coefficients. Again, the higher order behavior of the perturbative expansion is poorly known and we make no attempt to study the higher order contributions in the present work.

(3) Hadronic parameterizations:

This is the most challenging part in the sum rule analysis, because we have no ad hoc criterion to see how well our ansatz, e.g., the momentum independence of the excited state parameters E_A^N, E_B^N, works. There are some analyses which show that this part could bring in a large uncertainty to a sum rule calculation [19].

(4) The extraction of physical quantities from the matching:

Since physical quantities should be independent of the Borel mass, the deviation of the physical observables from a constant value in a given Borel window should not be larger

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[23]In presenting these figures, we have chosen the value of the $\bar{\theta}$ parameter to be 10^{-10}.

than the uncertainty associated with the choice of a Borel window. Nevertheless, the scheme dependent errors (how do we extract physical quantities from the matching) are related to the χ^2 of the matching, and represent the quality of a sum rule calculation.

(5) The dependence on the input parameters:

The QCD sum rule calculations generally rely on input parameters which are derived from other sources, e.g., the quark mass, quark condensates etc.. In many cases, these numbers are not well determined and their variations from a "standard" value have to be taken into account when we estimate the errors of the physical quantities from a QCD sum rule calculation. Recently, an analysis was performed by D.B. Leinweber [23] and we shall not dwell on this aspect further in this work.

4. Summary of the Results

In summary, we obtain the following results for the hadronic observables from the sum rule analysis:

$$M_N = 0.9 GeV \tag{123}$$

$$\tilde{\lambda}_N^2 = 2.5 GeV^6 \tag{124}$$

$$F_2^p = 3.1 \qquad\qquad F_2^n = -2.1 \tag{125}$$

$$F_3^p = 10^{-2}\bar{\theta} \qquad\qquad F_3^n = -0.5 \times 10^{-2}\bar{\theta} \tag{126}$$

VII. SUMMARY AND CONCLUSION

The electric dipole moments of nucleons (NEDM, d_N) are calculated using the method of QCD sum rules. Through the use of a polar representation of both nucleon EM moments (anomalous magnetic moment F_2^N and electric dipole moment F_3^N) and the $U_A(1)$ covariant quark condensates, we are able to demonstrate the reparameterization invariance of CP violating observables explicitly in a QCD Lagrangian with a CP violating $\bar{\theta}$ parameter. The symmetry constraints on strong CP violation in QCD, together with a dual relationship between the quark mass m_q and the chiral radius R_q [9] are realized in the QCD sum rule relations which connect the hadronic observables to the QCD parameters. The extraction of the NEDM in terms of the $\bar{\theta}$ parameter and QCD parameters from the QCD sum rule relations can be achieved without assuming a perturbative expansion of the $\bar{\theta}$ parameter. In addition, the periodic dependence of the CP violating observables on the $\bar{\theta}$ parameter comes out naturally in our approach.

Our final result establishes a functional dependence of the NEDM on the $\bar{\theta}$ parameter, which, combined with the experimental upper bound on the nEDM, gives an upper bound on the unknown $\bar{\theta}$ parameter of less than 10^{-9}. While this result is compatible with previous calculations of the nEDM using different techniques, it also indicates that a dynamical suppression of the CP violating observables, as implied by the symmetry constraints on the strong CP violation in QCD, is not sufficient to resolve the strong CP problem. Therefore, we conclude that a solution to the strong CP problem does not exist within QCD and a natural explanation of the tiny $\bar{\theta}$ parameter necessarily requires physics beyond the standard model.

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FIGURES

FIG. 1. Feynman Diagrams which contribute to the $\hat{p}$ sum rule

FIG. 2. Feynman Diagrams which contribute to the 1 and γ_5 sum rules

FIG. 3. Feynman Diagrams which contribute to the $\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_\beta\gamma_5$ sum rule

FIG. 4. Feynman Diagrams which contribute to the $\hat{p}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)(1, \ \gamma_5)$ sum rule

FIG. 5. The nucleon mass M_N as a function of the squared Borel mass M_B^2

FIG. 6. The nucleon residue $\tilde{\lambda}_N^2$ as a function of the squared Borel mass M_B^2

FIG. 7. The nucleon magnetic moment $F_1^N + F_2^N$ as a function of the squared Borel mass M_B^2

FIG. 8. The nucleon electric dipole moment F_3^N as a function of the squared Borel mass M_B^2

TABLES

TABLE I. Nucleon to higher state EM transition matrix element

(a)

(b)

(c)

FIG.1

(a)   (b)

FIG.2

(a)

(b)

FIG.3

(c)

36

(a)

(b)

FIG.4

(c)

FIG.5

FIG.6

FIG.7

NEDM x 10^(-12)

1.5

p

1

0.5

0

-0.5

n

-1

0.9   0.95   1   1.05   1.1   1.15   1.2

Squared Borel Mass MB^2
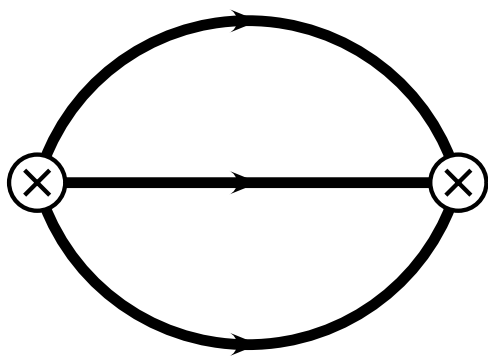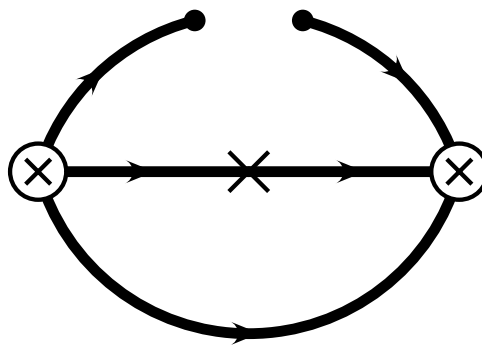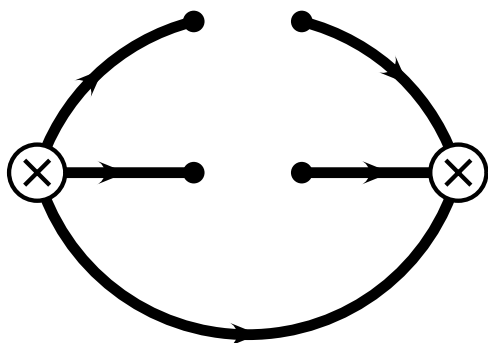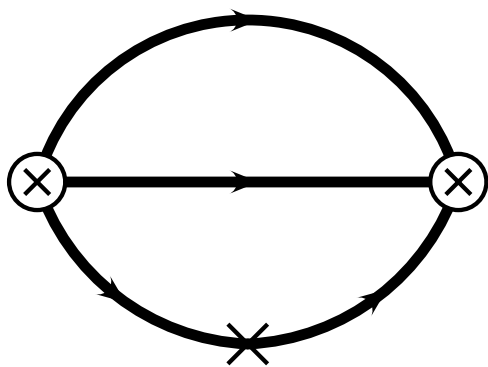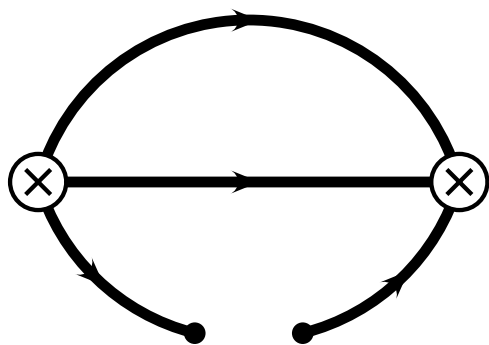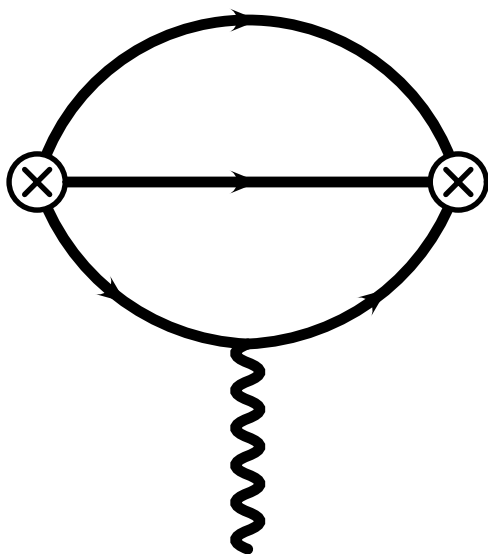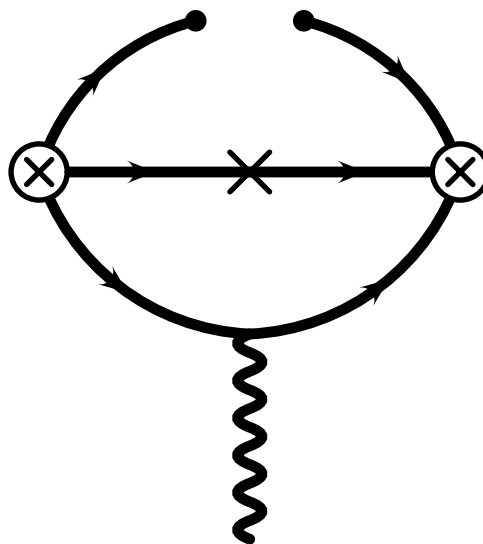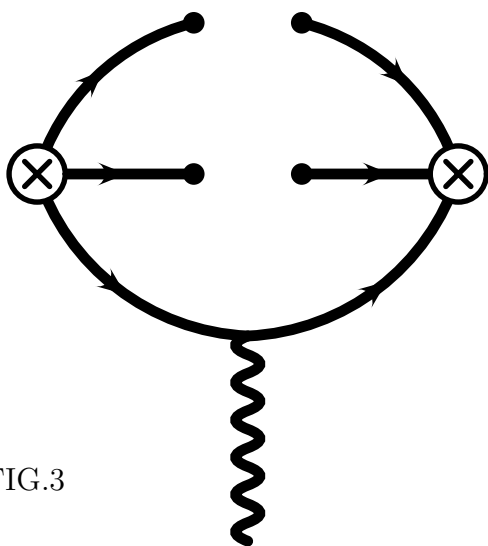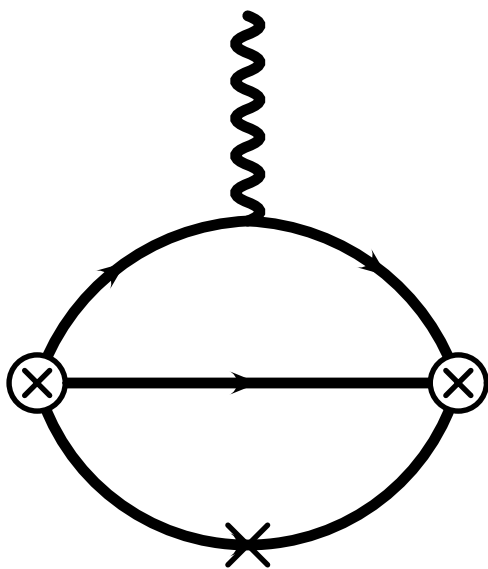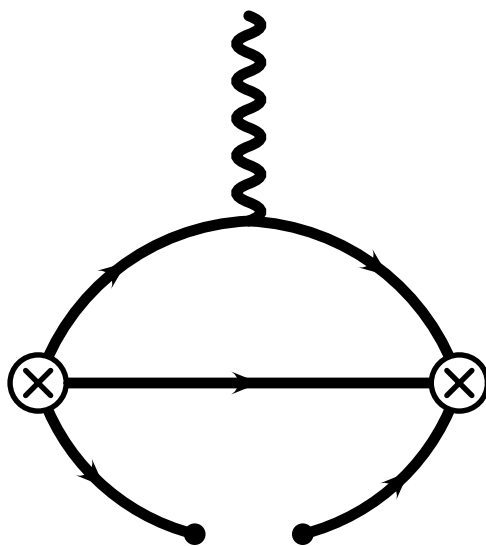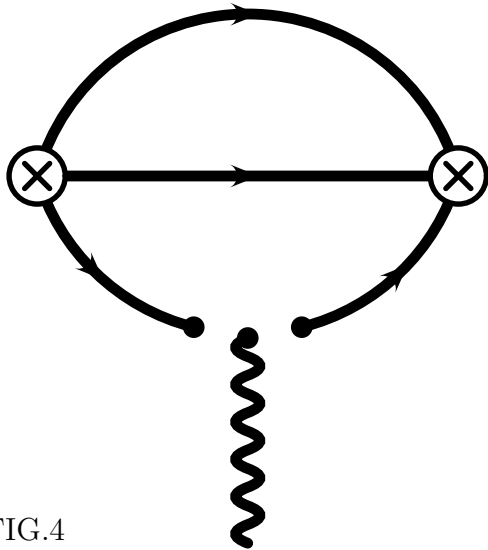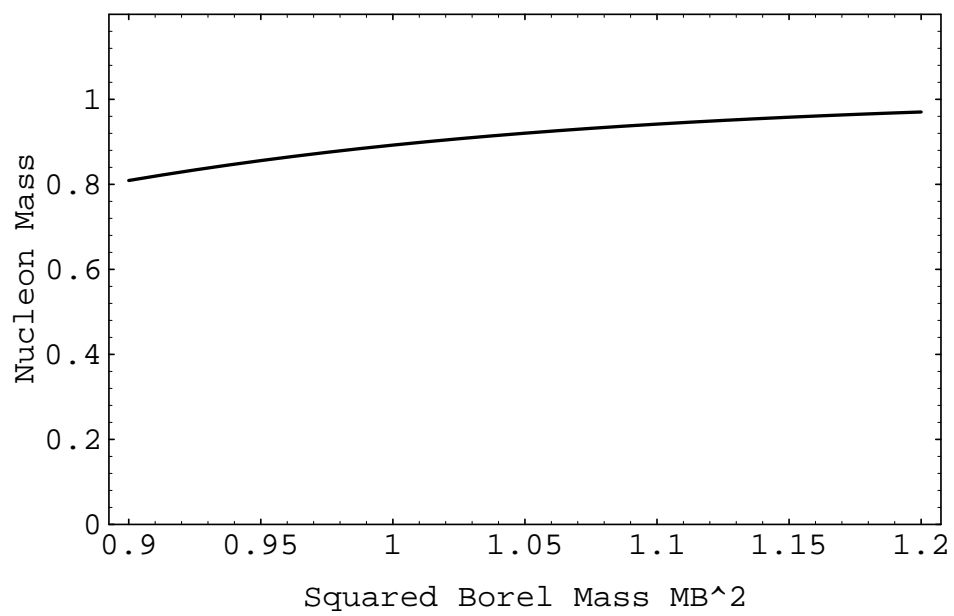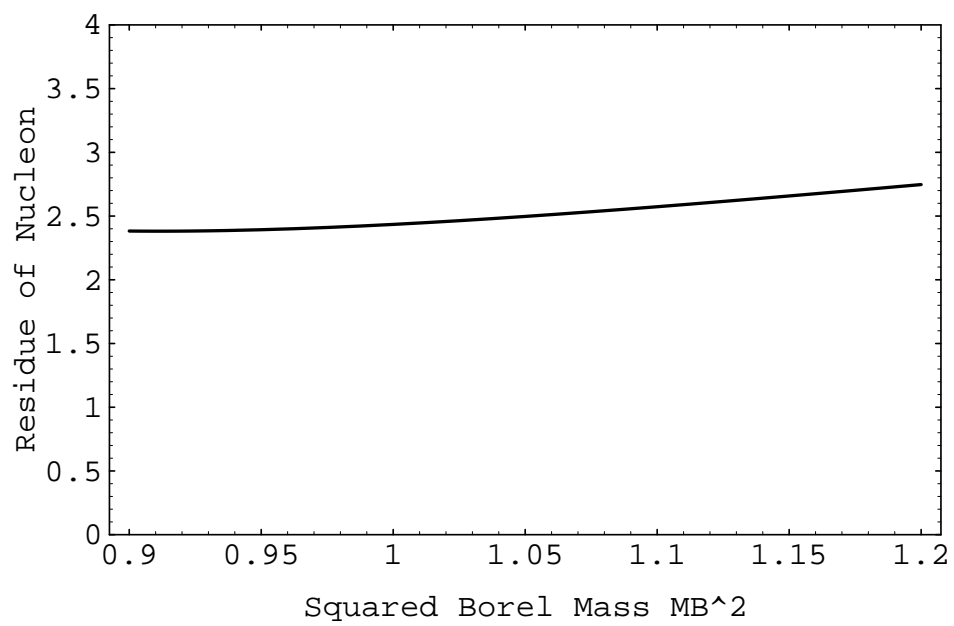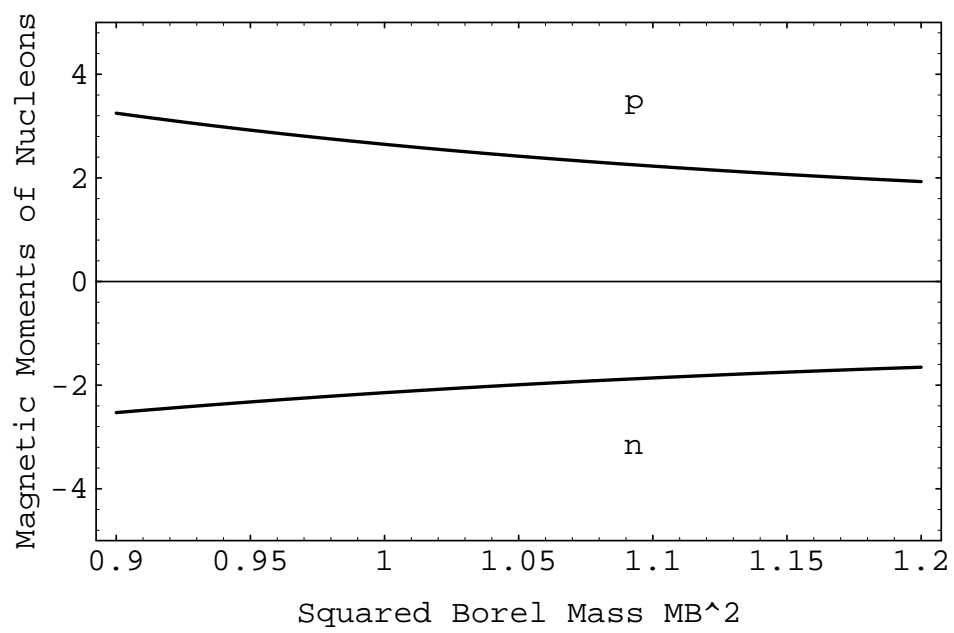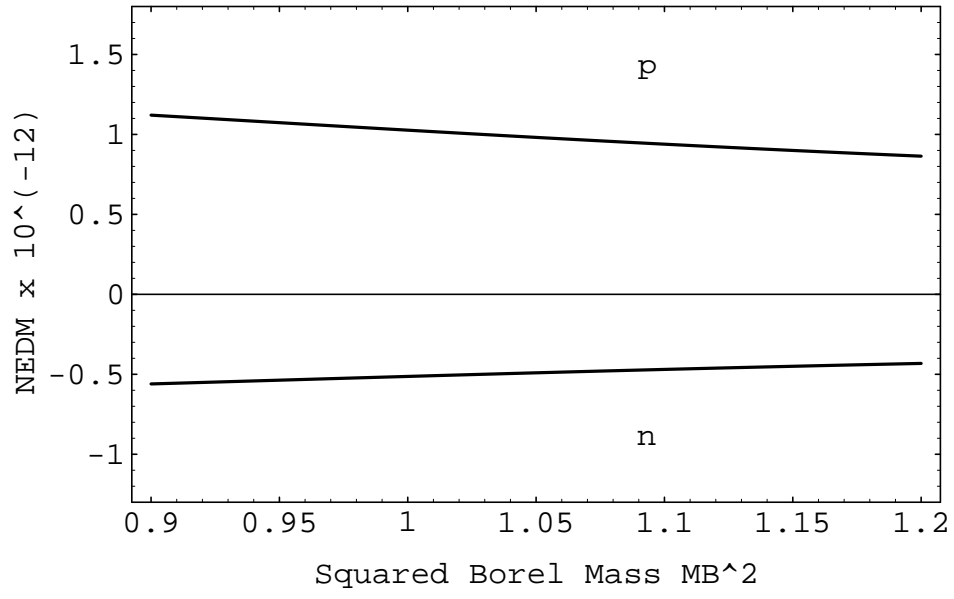
FIG.8

$\Pi_N^{\mu\nu}$	N	N''
N	$\langle N\vert J_\mu\vert N\rangle_{\theta_q,\theta_G}$ $Region1$	$\langle N\vert J_\mu\vert N''\rangle_{\theta_q,\theta_G}$ $Region2$
N'	$\langle N'\vert J_\mu\vert N\rangle_{\theta_q,\theta_G}$ $Region2$	$\langle N'\vert J_\mu\vert N''\rangle_{\theta_q,\theta_G}$ $Region3$

TABLE I